Induced Representations, Reproducing Kernels and the Conformal Group

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Abstract. The properties of "induced" (or multiplier) representations of groups which act in Hilbert spaces with a reproducing kernel are investigated. A resumé of earlier work is followed by a discussion of criteria for the irreducibility of such representations. The notions of reproducing kernel and positive definite spherical function are found to overlap. As a result, functional equations (analogous to those of Godement for spherical functions) are found for the reproducing kernel. The abstract theory is illustrated by certain discrete series representations of the conformal group and by their "limit points". In particular the so-called ladder representations (which give rise to the conformal symmetry of zero mass particles) are analysed from this viewpoint.

Reproducing Kernels

1. Introduction

This paper is mainly concerned with developing some results on group representations in Hilbert spaces with reproducing kernels. The ideas were motivated by the papers of Ruhl [2] on the conformal group, Bargmann's basic work [3] on the commutation relations and the paper of Perelomov [4] in which he points out that the notion of "coherent state" and reproducing kernel are the same.

The theory has its origins in the papers of Krein [9] and Aronszajn [10] although the foundations for the ideas developed here were laid by Kunze [1]. Subsection 2.1 is devoted to a resume of the results of [1]. This is followed by a discussion of criteria for the irreducibility of "induced representations" in reproducing kernel Hilbert spaces. The rest of Section 2 is concerned with the relationship between spherical functions and reproducing kernels. The existence of functional equations for reproducing kernels and their relationship to the functional equations satisfied by spherical functions of height one (see Godement [5] for this terminology) is investigated.

The general theory is illustrated by using the work of Graev [6] and Ruhl [2] on the discrete series representations of the conformal group [which for our