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Symmetry Conservation and Integrals over Local Charge Densities in Quantum Field Theory

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Abstract. For conserved local currents $\partial^\mu j_\mu(x)=0$ in quantum field theory it is shown that an R-dependence of $\alpha_R(x_0)$ in $j_0(f_R(\mathbf{x})\cdot\alpha_R(x_0))$ leads to nicer properties than a fixed $\alpha(x_0)$. The behaviour of $\lim_{R\to\infty}\|j_0(f_R(\mathbf{x})\cdot\alpha_R(x_0)\Omega\|$ is discussed under this aspect.

1. Introduction

In connection with symmetry transformations space integrals over the time component of divergenceless current densities are often considered in relativistic quantum field theory, more rigorously, one smears the current with appropriate testfunctions (see for instance [1] to [6] and further references there). In the following the Wightman framework will be assumed. Let us assume that the global charge operator Q exists with $Q\Omega=0$, where Ω is to be the unique vacuum vector, that means the symmetry is conserved, then the question arises, in what sense the sequence of local charge operators $Q_R:=j_0(f_R(\mathbf{x})\cdot\alpha(x_0))$ converges to the global one. If the time smearing is kept fixed, there is in general only a weak convergence on a dense set of states (the local or quasilocal ones) ([2, 5]). Furthermore there is no hope to prove weak convergence, even if a massgap is assumed, because of the divergence of $\|Q_R\Omega\|$. Moreover there is a close connection between the behavior of $\|Q_R\Omega\|$ and symmetry conservation. If the existence of Q is not a priori assumed, then $\lim_{R\to\infty}\|Q_R\Omega\|<\infty$ entails symmetry conservation ([7]). It is well known that the rate of divergence of $\|Q_R\Omega\|$ depends on the sequence of testfunctions $\{f_R(\mathbf{x})\}$ ([2, 3]). In [4] it was proved that in general $\|Q_R\Omega\|$ always diverges whatever sequence $\{f_R\}$ has been chosen. But there are important examples where this is not true (for instance tensorcurrents of second rank like the energy-momentum tensor [8]). We have proved a generalisation which gives optimal results and which shows that there are examples with $\|Q_R\Omega\|$ bounded in R. The results hold true irrespectively of the existence of a gap.

Let $g(\mathbf{p})$ denote $\int\tilde{G}(p)|\tilde{\alpha}(p_0)|^2dp_0$ where $\tilde{G}(p)$ is the Fouriertransform of $G(x-y):=(\Omega|j(x)j(y)\Omega)$.