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High-Temperature Analyticity in Classical Lattice Systems

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Abstract. We prove analyticity of the correlation functions for classical lattice systems, including "continuous-spin" systems, at high temperatures and in strong external fields. For systems whose configuration spaces are homogeneous spaces for compact groups (e.g. Ising, plane rotator and classical Heisenberg models), improved estimates on the region of analyticity are obtained by generalizing an integral equation of Gruber and Merlini. Exponential cluster properties are also obtained for such systems with a finite-range interaction.

I. Introduction

Using an equation of the "Kirkwood-Salsburg" type, Gallavotti and Miracle [2] have shown that the correlation functions and thermodynamic pressure of a classical lattice gas are analytic in the interaction parameters at high temperature and low activity. A similar equation can be used to deal with models where the "spin" at each site can take on any finite number of values, as opposed to the two values 0 and 1 for the lattice gas. However, this method does not extend to "continuous-spin" models.

In this paper, we present two models of proving analyticity for more general classical lattice systems. The first (Section II) works for very general systems, and is based on a theorem of Dobrushin [1] on uniqueness of measures with given conditional probabilities. The second method (Section III) requires some additional structure on the configuration space at each site: it must admit a transitive action by a compact group of measure-preserving homeomorphisms. We can then define an integral equation, generalizing one considered by Gruber and Merlini [3] for the spin $-\frac{1}{2}$ model. This involves a "Fourier series" expansion of the correlation functions; for a classical Heisenberg model, it is an expansion in spherical harmonics. In Section IV we obtain exponential cluster properties for finite-range interactions from this equation.