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Global Properties of Radial Wave Functions in Schwarzschild's Space-Time

II. The Irregular Singular Point

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Abstract. Two solutions $\mathscr{R}_5(x, x_s)$ and $\mathscr{R}_6(x, x_s)$ related to the irregular singular point at $x=+\infty$ of the radial wave equation in Schwarzschild's space-time are studied as functions of the independent variable x and the parameter x_s. Analytic continuations of $\mathscr{R}_5$ and $\mathscr{R}_6$ are derived and their relation to the flat-space case solutions is established. Explicit expressions for $\mathscr{R}_3(x, x_s)$ and $\mathscr{R}_4(x, x_s)$ (the solutions about the regular singular point at $x=x_s$) are given. From these expressions and the analytic continuations of $\mathscr{R}_5$ and $\mathscr{R}_6$ the coefficients relating linearly $\mathscr{R}_5$ and $\mathscr{R}_6$ with $\mathscr{R}_i$ $(i=1, 2, 3, 4)$ are calculated.

1. Introduction

The behavior of weak fields (scalar, electromagnetic or gravitational) around a Schwarzschild black hole is governed by a linear partial differential equation of second order. After separation of the angular variables and the time the resulting linear ordinary second order differential equation has two regular singular points at $x=0$ and $x=x_s$ and one irregular singular point at $x=+\infty$. The solutions of this differential equation cannot be expressed in terms of any known function of mathematical physics and very few properties of them are known. Thus numerical analysis is introduced sooner or later in the study of wave phenomena around black holes. This situation has been presented in more detail in a previous paper [1], which hereafter will be refered to as paper I. The objectives set in that paper can be described briefly as follows:

(a) Find analytic continuations of the six solutions $\mathscr{R}_i$ $(i=1,\ldots, 6)$ defined by their expansions at the singular points $x=0$, $x=x_s$, and $x=+\infty$.

(b) Relate the solutions $\mathscr{R}_i$ of the curved-space case to the solutions of the flat-space case (the spherical Bessel functions).

(c) Determine the analytic expressions of the coefficients $K_{ij}(x_s)$ which relate linearly any three of the solutions $\mathscr{R}_i$.

In paper I we examined four solutions $\mathscr{R}_i$ $(i=1,2, 3, 4)$ defined by their converging power series expansions about the regular singular points $x=0$ and $x=x_s$.