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Statistical Mechanics of Systems of Unbounded Spins

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Abstract. We develop the statistical mechanics of unbounded n-component spin systems on the lattice Z^ν interacting via potentials which are superstable and strongly tempered. We prove the existence and uniqueness of the infinite volume free energy density for a wide class of boundary conditions. The uniqueness of the equilibrium state (whose existence is established in general) is then proven for one component ferromagnetic spins whose free energy is differentiable with respect to the magnetic field.

1. Introduction

The study of continuous unbounded spin systems on a lattice has received great impetus in recent years from its close connection with Euclidian quantum field theory [1, 2]. While the applications to field theory require the passage to the limit of zero lattice spacing which poses great difficulties (yet to be overcome for the physically interesting situations) the lattice results are of interest in their own right and many of them carry over, more or less directly, to field theory once the existence of the latter is proven. Indeed certain lattice results are very helpful in proving the existence of the corresponding field theory.

In this paper we develop the general statistical mechanical theory of such systems: making use of what was done for continuum particle systems and Ising systems in the last decade [3, 4]. For this reason we consider interaction potentials of fairly long range and not necessarily of ferromagnetic type, although these are not currently of interest in field theory. We do however restrict ourselves essentially to pair-wise interactions; all higher spin interactions would have to be bounded. More general many spin interactions can be dealt with by an extension of those methods [5] but they will not be considered here.

Our main results are: a) the existence and uniqueness (independence of boundary conditions) of the infinite volume free energy density F, b) the existence of infinite volume "regular" equilibrium states as limits of finite volume Gibbs

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