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Probability Estimates for Continuous Spin Systems

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Abstract. Probability estimates for classical systems of particles with superstable interactions [1] are extended to continuous spin systems.

1. Notation and Assumptions

On a lattice $\mathbb{Z}^\nu$ we consider continuous d-dimensional spins. A *spin configuration* in $\Lambda \subset \mathbb{Z}^\nu$ is thus a function $s_\Lambda : \Lambda \mapsto \mathbb{R}^d$; its value at $x \in \Lambda$ will be denoted by s_x.

If $x = (x^1, \ldots, x^\nu) \in \mathbb{Z}^\nu$, we write $|x| = \max_i |x^i|$. If $s = (s^1, \ldots, s^d) \in \mathbb{R}^d$, we write $|s| = \left(\sum_i (s^i)^2\right)^{1/2} = \sqrt{s^2}$.

A measure $\mu \geqq 0$ on $\mathbb{R}^d$ is given such that

$$\int \mu(ds) e^{-\alpha s^2} < +\infty$$

if $\alpha > 0$, and μ is not identically 0.

We shall call *interaction* a real function U on all configurations in all finite $\Lambda \subset \mathbb{Z}^\nu$ satisfying the following conditions.

(a) U is $\otimes^\Lambda \mu$-measurable on $(\mathbb{R}^d)^\Lambda$ and invariant under translations of $\mathbb{Z}^\nu$.

(b) *Superstability.* There exist $A > 0$, $C \in \mathbb{R}$ such that if $s_\Lambda \in (\mathbb{R}^d)^\Lambda$ is a configuration on any finite Λ, then

$$U(S_\Lambda) \geqq \sum_{x \in \Lambda} [A s_x^2 - C] .$$

(c) *Regularity.* There exists a decreasing positive function Ψ on the natural integers such that

$$\sum_{x \in \mathbb{Z}^\nu} \Psi(|x|) < +\infty .$$

Furthermore if Λ_1, Λ_2 are disjoint finite subsets of $\mathbb{Z}^\nu$ and s_{Λ_1}, s_{Λ_2} the restrictions to Λ_1, Λ_2 of a configuration $s_{\Lambda_1 \cup \Lambda_2}$ on $\Lambda_1 \cup \Lambda_2$, then

$$|W(s_{\Lambda_1 \cup \Lambda_2})| \leqq \sum_{x \in \Lambda_1} \sum_{y \in \Lambda_2} \Psi(|y-x|) \tfrac{1}{2} (s_x^2 + s_y^2)$$