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A New Method for Constructing Factorisable Representations for Current Groups and Current Algebras

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Abstract. Let $C_e^\infty(R^n, G)$ denote the group of infinitely differentiable maps from n-dimensional Euclidean space into a simply connected and connected Lie group, which have compact support. This paper introduces a class of factorisable unitary representations of $C_e^\infty(R^n, G)$ with the property that the unitary operator U_f corresponding to a function f in $C_e^\infty(R^n, G)$ depends not only on f, but also on the derivatives of f up to a certain order. In particular these representations can not be extended to the group of all continuous functions from R^n to G with compact support.

§ 1. Introduction

Let G be a simply connected and connected Lie group and let $\mathscr{G}$ be its Lie algebra. Let $\exp:\mathscr{G}\to G$ denote the exponential map. We denote by $C_e^\infty(R, G)$ the class of all C^∞ maps from R into G with compact support. A map $\varphi: R\to G$ is said to have compact support if takes the value e, i.e., the identity element of G outside a compact set. Let $C_0^\infty(R, \mathscr{G})$ denote the class of all infinitely differentiable maps from R into the vector space $\mathscr{G}$ with compact support. For any $f\in C_0^\infty(R, \mathscr{G})$, we define $\operatorname{Exp} f\in C_e^\infty(R, G)$ by writing $(\operatorname{Exp} f)(x)=\exp f(x)$, for all $x\in R$. $C_e^\infty(R, G)$ is a group (under pointwise multiplication) and $C_0^\infty(R, \mathscr{G})$ is a Lie algebra (under pointwise addition, scalar multiplication and Lie brackets). These may respectively be called as current group and current algebra over R. We give $C_0^\infty(R, \mathscr{G})$ the usual Schwarz topology. A homomorphism $\varphi\to U_\varphi$ of the group $C_e^\infty(R, G)$ into the group of unitary operators on a Hilbert space H is said to be a *unitary representation* or simply a representation if $U_{\operatorname{Exp} f_n}$ converges weakly to $U_{\operatorname{Exp} f}$ whenever $f_n\to f$ as $n\to\infty$ in the topology of $C_0^\infty(R, \mathscr{G})$.

For any compact set $K\subset R$, let $C(K, G)\subset C_0^\infty(R, G)$ be the subgroup of all those maps with support contained in K. If K_1, K_2 are two disjoint compact subsets of R, $C(K_1\cup K_2, G)$ can be identified in a natural manner with the cartesian product $C(K_1, G)\times C(K_2, G)$. Indeed, for any $\varphi\in C(K_1\cup K_2, G)$, define

$$\begin{aligned}\varphi_i(x)&=\varphi(x) \quad \text{if} \quad x\in K_i\\ &=e \quad \text{if} \quad x\notin K_i, \quad i=1,2.\end{aligned}$$