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A Possible Constructive Approach to φ_4^4 III

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Abstract. We continue our investigation into a constructive approach for φ_d^4 and prove under the hypotheses of our previous work that on a lattice and in the single phase region the theory is uniquely determined by the intermediate renormalization conditions. Then the Gaussian theory and the Ising model are the two extremal cases for all φ_d^4 theories.

In the first of two papers devoted to a constructive approach to φ_4^4 in the single phase region ([4, 5]) we presented an ansatz based on an implicit function problem and suggested by multiplicative renormalization. The mapping is (within the Euclidean approach on a fixed lattice) from the set of the three renormalization constants to the set of the three (intermediate) normalization constants. We will call this map the renormalization map. In Ref. [4] we made the assumption that this C^∞-map is everywhere locally injective (i.e. of maximal rank). Using this assumption and a certain assumption about the image at infinity (see below), we will prove that this map is actually globally injective, i.e. one-to-one. Thus the normalization constants uniquely determine the theory in the lattice approximation.

The strategy for the proof will be as follows: First we show that a certain set which we showed, in Ref. [4], to be in the image is actually all of the image. In particular the image is simply connected. This result also allows one to view the Ising model as one extremal case of a φ_d^4 theory on a lattice: It is the case where $\varphi^2 = \text{const.}$ or equivalently where the bare coupling constant tends to infinity. This conforms with conventional wisdom, see e.g. Ref. [1, 2, 7]. Secondly we show that the renormalization map is proper, thus making the set of renormalization constants a covering space of the set of normalization constants. The global injectivity then follows from a standard theorem in homotopy theory.

To fix the notation, we start by reviewing the results of Ref. [4].

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