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The Cluster Expansion in Statistical Mechanics*

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Abstract. The Glimm-Jaffe-Spencer cluster expansion from constructive quantum field theory is adapted to treat quantum statistical mechanical systems of particles interacting by finite range potentials. The Hamiltonian H_0+V need be stable in the extended sense that $H_0+4V+BN\geqq 0$ for some B. In this situation, with a mild technical condition on the potentials, the cluster expansion converges and the infinite volume limit of the correlation functions exists, at low enough density. These infinite volume correlation functions cluster exponentially. We define a class of interacting boson and fermion particle theories with a matter-like potential, $1/r$ suitably truncated at large distance. This system would collapse in the absence of the exclusion principle—the potential is unstable—but the Hamiltonian is stable. This provides an example of a system for which our method proves existence of the infinite volume limit, that is not covered by the classic work of Ginibre, which requires stable potentials.

One key ingredient is a type of Holder inequality for the expectation values of spatially smeared Euclidean densities, a special interpolation theorem. We also obtain a result on the absolute value of the fermion measure, it equals the boson measure.

1. Introduction

In the quantum statistical mechanical theory of matter (positive charged particles and negative charged identical fermions interacting with a $1/r$ potential) the most basic result is the stability, first proved by Dyson and Lenard in [2]. One of the authors presented a new proof in [3], and recently another proof was given by Lieb and Thirring in [8]. The second basic result was the proof of the existence of the thermodynamic functions in the infinite volume, by Lieb and Lebowitz in [7]. A natural next problem is the existence of the infinite volume correlation functions, for some range of parameters—an open question.

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