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Generalized *K*-Flows

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Abstract. The classical concept of *K*-flow is generalized to cover situations encountered in non-equilibrium quantum statistical mechanics. The ergodic properties of generalized *K*-flows are discussed. Several non-isomorphic examples are constructed, which differ already in the type (II_1, III_λ, and III_1) of the factor on which they are defined. In particular, generalized factor *K*-flows with dynamical entropy either zero (singular *K*-flows) or infinite (special non-abelian *K*-flows) are constructed.

Introduction

The *motivation* for this paper stems from the following schematic description of the *purpose* of non-equilibrium statistical mechanics.

Given a *dissipative, thermodynamical* system $\{\mathfrak{N}_S, \phi_S, \gamma(\mathbb{R}^+)\}$, devise: (i) a thermal bath $\{\mathfrak{N}_R, \phi_R\}$, and (ii) an interaction between $\mathfrak{N}_S$ and $\mathfrak{N}_R$, in such a manner that the following conditions be satisfied. Firstly, the composite dynamical system $\{\bar{\mathfrak{N}} = \mathfrak{N}_S \otimes \mathfrak{N}_R, \bar{\phi} = \phi_S \otimes \phi_R, \bar{\alpha}(\mathbb{R})\}$ should be *conservative*, and understandable from the laws of hamiltonian mechanics. Secondly, $\gamma(\mathbb{R}^+)$ should appear as the *restriction*, to the system $\mathfrak{N}_S$ of interest, of the total evolution $\bar{\alpha}(\mathbb{R})$; namely, for every (normal) state ψ on $\mathfrak{N}_S$, every observable N in $\mathfrak{N}_S$, and all positive times t, one should have:

$$\langle \psi \otimes \phi_R ; \bar{\alpha}(t)[N \otimes I] \rangle = \langle \psi ; \gamma(t)[N] \rangle . \tag{1}$$

To be specific, we shall assume that ϕ_S and ϕ_R are thermal equilibrium states, respectively for the von Neumann algebras $\mathfrak{N}_S$ and $\mathfrak{N}_R$. In line with the ideas of non-equilibrium thermodynamics, we shall further assume that $\gamma(\mathbb{R}^+)$ is a semi-group of positive, linear maps of $\mathfrak{N}_S$ into itself such that $\phi_S \circ \gamma(t) = \phi_S$ for every $t \in \mathbb{R}^+$, and that $\langle \psi ; \gamma(t)[N] \rangle$ approaches $\langle \phi_S ; N \rangle$ when t tends to $+\infty$, for every

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