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The Existence of Maximal Slicings in Asymptotically Flat Spacetimes

M. Cantor*, A. Fischer** ***, J. Marsden+ ***, N. Ō Murchadha++, and J. York++

Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27514, USA

Abstract. We consider Cauchy data (g, π) on $\mathbb{R}^3$ that are asymptotically Euclidean and that satisfy the vacuum constraint equations of general relativity. Only those (g, π) are treated that can be joined by a curve of "sufficiently bounded" initial data to the trivial data $(\delta, 0)$. It is shown that in the Cauchy developments of such data, the maximal slicing condition $\operatorname{tr}\pi = 0$ can always be satisfied. The proof uses the recently introduced "weighted Sobolev spaces" of Nirenberg, Walker, and Cantor.

Consider the set $\mathscr{C}$ of spacetimes which are the Cauchy developments of initial data (g, π) on $\mathbb{R}^3$ which are asymptotically Euclidean and which satisfy the constraint equations [see (3) and (4) below] in the dynamical formulation of general relativity [1]. In 1968, Brill and Deser [2] conjectured that one can maximally slice any such spacetime, i.e. one can find spacelike hypersurfaces on which $\operatorname{tr}\pi = 0$. In a Hamiltonian analysis of general relativity $\operatorname{tr}\pi$ assumes the role of a gauge variable (see for example [12]) and so one would expect that the $\operatorname{tr}\pi = 0$ condition can be met in any such spacetime. Here we prove that the Brill-Deser conjecture is true.

We consider only those (g, π) which can be joined by a curve of "sufficiently bounded" initial data (to be explained later) to flat space $(\delta, 0)$. Thus we are considering the component $\mathscr{C}_0$ of $(\delta, 0)$ in the set of asymptotically Euclidean solutions of the constraint equations. $\mathscr{C}_0$ is restricted to those 3-metrics which are derived from Lorentz metrics on $\mathbb{R}^4$ that are near the "background" Minowski metric. The set $\mathscr{C}_0$ is discussed in [7–11].

* Present address: Department of Mathematics, Duke University, Durham, North Carolina 27706, USA

** Present address: Department of Mathematics, University of California, Santa Cruz, California 95064, USA

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+ Present address: Department of Mathematics, University of California, Berkeley, California 94720, USA

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