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Statistical Geometry and Space-Time

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Abstract. In this paper I try to construct a mathematical tool by which the full structure of Lorentz geometry to space time can be given, but beyond that the background – to speak pictorially – the subsoil for electromagnetic and matter waves, too. The tool could be useful to describe the connections between various particles, electromagnetism and gravity and to compute observables which were not theoretically related, up to now. Moreover, the tool is simpler than the Riemann tensor: it consists just of a set S of line segments in space time, briefly speaking.

The System S

We first consider a global euklidian structure in the fourdimensional real number space $\mathbb{R}^4$. Later on, also local structures and curvature will be taken into account.

Let us denote the points of $\mathbb{R}^4$ by $\mathfrak{x}=(x_0, \ldots, x_3)$, $\mathfrak{y}=(y_0, \ldots, y_3)$ and so on. The first component of the quadruples always is called the time coordinate. We put $\mathfrak{x}'=(x_1, \ldots, x_3)$, $\mathfrak{y}'=(y_1, \ldots, y_3)$ and so forth. By $\delta(\mathfrak{x}, \mathfrak{y})$ a Radon measure in $\mathbb{R}^4 \times \mathbb{R}^4$ is denoted which has the union of (full) light cones for support: $L=\{(\mathfrak{x}, \mathfrak{y})=(\mathfrak{y}-\mathfrak{x})^2=0\}$, where $\mathfrak{z}^2=-z_0^2+z_1^2+z_2^2+z_3^2$ is the Lorentz norm. If $\mathfrak{f}$ is a test function in $\mathbb{R}^4 \times \mathbb{R}^4$ then the following equation shall be valid:

$$\int\limits_{\mathbb{R}^4 \times \mathbb{R}^4} \delta(\mathfrak{x}, \mathfrak{y})\mathfrak{f}(\mathfrak{x}, \mathfrak{y})=\int\limits_{L} d\mathfrak{x}d\mathfrak{y}' \, \|\mathfrak{y}'-\mathfrak{x}'\|^{-1}\mathfrak{f}(\mathfrak{x}, \mathfrak{y}) .$$

Here $\|\mathfrak{y}'-\mathfrak{x}'\|$ is the euclidean distance of $\mathfrak{y}'$ and $\mathfrak{x}'$ in $\mathbb{R}^3$. It is well known that the measure is invariant under all translations and all Lorentz-transformations (that means under all transformations of the Poincaré group). Furthermore it is symmetric in $\mathfrak{x}$ and $\mathfrak{y}$ and on the other hand a linear transformation which leaves it invariant belongs to the Poincaré group.

To prevent misunderstanding: I am far from asserting to have really solved any physical problem. This short paper gives some mathematical ideas, only, which might – I hope – prove to be helpful in future