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A Possible Constructive Approach to ϕ_4^4

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Abstract. We propose a constructive approach to ϕ_4^4. It is based on formulating the ϕ_4^4 theory as an implicit function problem using multiplicative renormalization. For the corresponding lattice formulation we reduce the problem to verifying three conjectures. One conjecture is a regularity condition. The remaining two concern properties of the classical Ising ferromagnet, one of which we discuss in the frame work of critical point analysis.

I. The Approach (Formal Considerations)

In recent years constructive field theory has made tremendous progress by using euclidean methods (see e.g. [12, 13, 22], and the literature quoted there). However, so far only superrenormalizeable theories have been successfully treated, since the techniques involved mostly rely on additive renormalization. In this article we propose the use of multiplicative renormalization. We have the philosophy respectively the rigourous result in mind that in perturbation theory additive renormalization, multiplicative renormalization and the BPHZ formulation are equivalent (see e.g. [14, 23]). Now in the ϕ_4^4 theory there are three renormalization constants entering the multiplicative renormalization procedure

(i) the mass counterterm δm^2;
(ii) the amplitude renormalization constant $Z_3 \geqq 0$;
(iii) the vertex function renormalization constant $Z_4 \geqq 0$.

On the other hand, there are three normalization conditions for the theory. Two involve the point function and one the four point function. Our central idea is simply to try to determine the renormalization constants for given normalization constants. Now usually the relativistic two point function is normalized by requiring a pole with residue 1 at (relativistic) $p^2 = m^2 > 0$. Since we are interested in formulating and solving the theory in the euclidean framework, we will instead work with the intermediate renormalization [2] or more precisely a generalization of it. There the two point function is normalized at $p^2 = 0$. We note that in perturbation theory, it is irrelevant, where the normalization is done (see e.g. [14]).