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The Classical Limit for Quantum Dynamical Semigroups

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Abstract. We describe a class of single-particle quantum-mechanical dynamical semigroups which, in the classical limit, give rise to Markov semigroups on phase space.

§ 1. Introduction

The close connection between quantum-mechanical dynamical semigroups and Markov semigroups has been considerably clarified recently. Both are particular cases of an abstract theory of stochastic processes [1, 2] and the latter can also arise from the former by restricting to a special class of states called quasi-classical or coherent states [3, 11]. As a new aspect of the connection we show that one obtains Markov semigroups by taking the classical limit of certain dynamical semigroups in a suitable manner. The dynamical semigroups we start with are of the type which arise in the weak or singular coupling limit of a quantum-mechanical particle interacting with an infinite free reservoir [4, 5, 8, 9, 12, 13, 15], but we do not pursue this here.

We take the evolution of an open system to be described by a strongly continuous one-parameter "dynamical" semigroup

$$T_\lambda(t) = \exp\{\lambda^{-2} Z + K)t\}. \tag{1.1}$$

on a Banach space V, called the state space. The unbounded operator Z is the generator of a strongly continuous one-parameter group of isometries e^{Zt} on V which determines the free evolution. The bounded operator K describes a perturbation of a "stochastic" type due to the influence of the external world. For reasons given in the references above we examine the asymptotic form of the evolution in the (weak or singular) coupling limit $\lambda \to 0$, where λ is real. In typical cases the effect of K integrated over all time is not finite, so the formalism of scattering theory is not appropriate. Moreover $T_\lambda(t)$ is generally a contraction only for $t \geqq 0$, so we restrict attention to such times t from now on.