Commun. math. Phys. 49, 53—61 (1976)

Communications in
Mathematical
Physics

Hamilton-Jacobi Separable, Axisymmetric, Perfect-fluid Solutions of Einstein's Equations

Sotirios Bonanos
N.R.C. Democritos, Aghia Paraskevi, Attiki, Greece

Abstract. In this paper we examine the Einstein equations with a perfect fluid source under the assumptions of (i) axial symmetry and time-independence, (ii) uniform rotation of the fluid about the symmetry axis, and (iii) separability of the Hamilton-Jacobi equation for the null geodesics of the space. These assumptions are made in an attempt to generalize the results of a similar investigation by Carter for the source-free case.
We first extend Carter's results by showing that his additional assumption of separability of the wave equation is unnecessary, it being a consequence of the field equations.
When the density of the fluid is non-zero, we are led to a particular solution discovered by Wahlquist, or to more symmetrical interior solutions with spherical equipressure surfaces. Except for the case of no rotation, these solutions cannot be matched to asymptotically flat exteriors.

1. Introduction

In a 1968 paper [1], Carter examined spaces with a two-parameter Abelian isometry group in which the Hamilton-Jacobi equation for the geodesics separates. While the first assumption (two-parameter Abelian isometry group) is readily interpreted physically as restricting the study to stationary and axisymmetric spacetimes, the second assumption ($H-J$ separability) has no simple physical justification. It is made, as Carter admits, in order to obtain sufficiently strong restrictions to make a detailed study possible.

In the same spirit, we use, in this paper, a weaker form of Carter's separability ansatz as a simplifying tool in the study of rotating fluid masses. We find that the only solution having an isometry group with fewer than 3 parameters is one discovered by Wahlquist starting from different assumptions [2]. We also obtain other solutions having spherical symmetry. None of these solutions, however, can represent a finite, isolated object in an asymptotically flat space, except in the limit of no rotation.