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On the Hartree-Fock Time-dependent Problem

A. Bove

Istituto di Fisica dell'Università di Bologna and Istituto Nazionale di Fisica Nucleare, Sezione di Bologna, I-40126 Bologna, Italy

G. Da Prato

Istituto Matematico "G. Castelnuovo" dell'Università di Roma, Roma, Italy

G. Fano

Istituto di Fisica dell'Università di Bologna and Istituto Nazionale di Fisica Nucleare, Sezione di Bologna, I-40126 Bologna, Italy

Abstract. A previous result is generalized. An existence and uniqueness theorem is proved for the Hartree-Fock time-dependent problem in the case of a finite Fermi system interacting via a two body potential, which is supposed dominated by the kinetic energy part of the one-particle hamiltonian.

1. Introduction

In this paper we consider the existence problem for the Hartree-Fock time-dependent equations of a finite system of fermions. This problem was first solved using fixed point theorems for local contractions in Banach spaces in Ref. [1], for the case of a bounded two body potential, and in Ref. [2][1] for the case of the repulsive Coulomb potential.

In the present paper we extend those results to a general potential, bounded from below and "essentially" dominated by the one-particle hamiltonian (for instance the laplacian operator). Our main result is Proposition 5.5., which proves the existence and uniqueness of a global solution, both in the case of the classical and of the mild solution, according to the smoothness of the initial data[2].

2. Notations and Hypotheses

We denote by:

E a Hilbert space with inner product $\langle\cdot,\cdot\rangle$;

[1] The paper [1] considers the case of arbitrary N and not only the case $N=2$ like erroneously stated in Ref. [2].

[2] While this work was in preparation, we received a preprint by Chadam and Glassey [3], where formal proofs have been obtained for the case of the Coulomb potential. Furthermore Definition 2.1. of [3] must be revised since the expression $\|K\|_{1,1}=\mathrm{Tr}(A|K|A)$ does not satisfy the triangle inequality.