

Positive Linear Maps on Operator Algebras

David E. Evans

School of Theoretical Physics, Dublin Institute for Advanced Studies, Dublin 4, Ireland

Abstract. Given a family of completely positive maps, indexed by a group, from a C^* -algebra into itself, we are concerned with its dilation to a group of $*$ -automorphisms on a larger algebra. A Schwarz-type inequality for n -positive $*$ -linear mappings from an involutive algebra into the bounded linear operators on a hilbert space is obtained. Strongly continuous one-parameter semigroups and groups on C^* -algebras, which have certain positivity properties, are studied.

1. Introduction

Given a family of completely positive maps, indexed by a group G , from a C^* -algebra into itself, we are concerned with its dilation to a group of $*$ -automorphisms on a larger algebra. Theorem 1 is an operator algebra analogue of B.Sz.-Nagy's hilbert space [12], and E.Stroescu's Banach space [11] dilations. This generalises to an arbitrary group some of the work of E.B.Davies [4] within the theory of quantum stochastic processes, and that of G.Lindblad [9] for quantum dynamical semigroups (both working in the Schroedinger picture).

We obtain a Schwarz-type inequality for certain n -positive mappings on involutive algebras. This produces some results for strongly continuous one-parameter semigroups and groups on C^* -algebras, which have various positivity preserving properties, e.g. 2-positivity, complete positivity. This extends the work of G.Lindblad [9] on the infinitesimal generators of norm continuous semigroups of completely positive maps. The general problem of characterising the infinitesimal generators of strongly continuous semigroups of completely positive maps is very difficult. As in the hilbert space situation, we expect that progress will be made in the other direction in obtaining techniques for constructing generators using more and more complicated unbounded dissipative and hamiltonian terms.

We remark that the physical significance of completely positive maps as operations on quantum systems has been observed in [8, 9].