Commun. math. Phys. 47, 229—231 (1976)

Communications in Mathematical Physics

On the Four-Valuedness of Twistors

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Abstract. The spinors on compactified Minkowski space, in terms of which twistor theory is formulated, are really U-spinors. In this light zero-mass fields have no Grgin discontinuity.

I shall examine the spinors which are induced on compactified Minkowsky space, M^c, by twistors. The notation will follow [3], to which the reader is referred for the basic facts of twistor theory. Note in particular that I shall mainly be using *concrete* indices[1], since the abstract index notation of [4] presupposes the existence of some particular spin structure; and it is precisely this that I wish to explore.

If Z and W are two twistors with components $(Z^\alpha)=(\eta^{\mathfrak{A}}, \iota_{\mathfrak{X}'})$, $(W^\alpha)=(\xi^{\mathfrak{A}}, \sigma_{\mathfrak{X}'})$, then they determine the point $x(Z, W)$ in Minkowski space M whose components are

$$x^a = -i\sigma^{a\mathfrak{A}\mathfrak{X}'}(\eta_{\mathfrak{A}}\sigma_{\mathfrak{X}'} - \xi_{\mathfrak{A}}\iota_{\mathfrak{X}'})/\iota_{\mathfrak{Y}'}\sigma^{\mathfrak{Y}'}\,, \tag{1}$$

provided that $\iota_{\mathfrak{Y}'}\sigma^{\mathfrak{Y}'} \neq 0$. Then an element g of the twistor transformation group SU (2, 2) [5] determines a local conformal transformation $\tilde{g}$ on M by

$$\tilde{g}(x(Z, W)) = x(g(Z), g(W))\,,$$

in a domain where both sides are defined.

The two pairs of numbers which make up the components of a twistor are interpreted on M as the components of spinors with respect to a fixed coordinate basis. Not only are they related to vectors by (1), but for any Poincaré transformation $\tilde{g}$ on M one can find a g which acts on these twistor components in the way appropriate to the spinor interpretation. Moreover, this action extends to conformal transformations, under which the $\iota_{\mathfrak{X}'}$ and $\eta^{\mathfrak{A}}$ transform as the components of spinors on M of conformal weight 1 (i.e. under dilatation by a factor θ they acquire a factor θ^{-1}). Hence they are describable in terms of the conformal metric alone, and so can be defined on the image of M in M^c. However, it is well known ([3],

[1] For typographical reasons concrete twistor indices are represented by α, β etc., instead of the Hebrew of [3].