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Necessary and Sufficient Conditions for Integral Representations of Wightman Functionals at Schwinger Points

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Abstract. Given a set of Wightman functions one would like to associate to it a field on Euclidean space admitting a simultaneous diagonalization. We investigate when this can be done in such a way that the Schwinger functions are the expectation values of this commutative field with a bounded metric operator commuting with the field. This requires as a tool the characterization of those linear functionals on the symmetric tensor algebra over a space of test functions which can be represented by complex measures on the corresponding space of distributions.

1. Introduction

The representation of Schwinger functions as moments of a measure on a space of distributions has, in the past few years, proved to be a useful tool in constructive quantum field theory. It is therefore interesting to ask to what extent such a representation can be derived from the Wightman axioms.

Starting from these axioms one can go by analytic continuation to the Schwinger points (points with real space and imaginary time components and no two arguments coinciding) and obtain the Wightman functions at Schwinger points. These functions are totally symmetric in their arguments but not defined at coinciding points. Therefore these Wightman functions at Schwinger points can be viewed as a linear functional on the symmetric tensor algebra of test functions, which is not everywhere defined but only on a subspace of this algebra. An extension of this functional to the whole algebra we will call a Schwinger functional.

By using the Hahn-Banach theorem one sees that such an extension is by no means unique. On the other hand this freedom does not affect the physics in the Minkowski world so that one can use it and try to enforce additional properties for convenience. We will try to find an extension such that the Schwinger functional is given by a (complex) measure. This property is automatically fulfilled by functionals on an abelian C^*-algebra, but not for general abelian algebras, due to