

# Remarks on the Relation between the Lee-Yang Circle Theorem and the Correlation Inequalities

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**Abstract.** We investigate the relation between the Lee-Yang circle theorem and the correlation inequalities. These results are general and independent of models. General properties of the partition functions which belong to the Lee-Yang class are given.

## 1. Introduction

Recently several authors have investigated the Euclidean boson quantum field models [the so-called  $P(\phi)_d$ -models] as a classical statistical mechanics [1, 2]. In these articles we see that the Lee-Yang circle theorem and the correlation inequalities play a central role in the study. On the other hand, Griffiths *et al.* conjectured that a set of correlation inequalities determine the forms of the interactions [3, 4]. From the points of view of these applications and conjectures, it is an interesting problem to decide the partition functions which satisfy the Lee-Yang circle theorem or the desired correlation inequalities.

Moreover, Newman recently proved that the Lee-Yang circle theorem leads to some correlation inequalities [5]. Therefore it is also an interesting problem to discuss the relation between the Lee-Yang circle theorem and the correlation inequalities. Finally we investigate the general properties of partition functions which satisfy the Lee-Yang circle theorem, as they have not appeared elsewhere.

We organize the paper as follows: In Section 2, we define classes of the partition functions  $\mathcal{P}_e, \mathcal{L}, \mathcal{D}, \mathcal{I}$ , and summarize the relevant correlation inequalities without proof. In Section 3, we investigate the Griffiths first (G-1) and the second (G-2) inequalities and discuss the relation between these inequalities and the Lee-Yang circle theorem. In Sections 4 and 5, we investigate the Griffiths-Hurst-Sherman inequality (GHS inequality) and the Lebowitz inequality. In Section 6, general properties of the partition functions which belong to the Lee-Yang class are given.