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Statistical Mechanics of a One-dimensional Lattice Gas with Exponential-polynomial Interactions

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Abstract. Some properties of the transfer-matrix for a one-dimensional classical lattice-gas with exponential-polynomial pair interactions are studied using Hilbert space techniques.

I. Introduction and Statement of Results

We are concerned here with the statistical mechanics of a classical, one-dimensional lattice-gas, or equivalently of a spin system with exponentially decreasing pair interactions of the type

$$\varphi_1(n)=\lambda^n \sum_{i=0}^{p} c_i n^i \qquad (0<\lambda<1) \tag{1.1}$$

as well as potentials which are a finite sum of decreasing exponentials,

$$\varphi_2(n)=\sum_{i=1}^{k} c_i \lambda_i^n \qquad (0<\lambda<1) \tag{1.2}$$

potential (1.1) will be termed exponential-polynomial type. Ruelle [1] [1] has established the absence of phase transitions in one-dimensional systems with translationally invariant two-body interactions that satisfy the condition

$$\sum_{i\in\mathbb{N}} i|\varphi(0,i)|<\infty \tag{1.3}$$

where $\mathbb{N}$ is the set of all integers >0.

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1 Ruelle's results actually extend to many-body translationally invariant interactions which satisfy the following criterion

$$\sum_{l>0} \sum_{0<i_1<i_2<\ldots<i_l} i_l|\varphi^{(l+1)}(0,i_1,i_2,\ldots,i_l)|<\infty$$

where $\varphi^{(l+1)}$ is the $(l+1)$ body potential.