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The Time-dependent Hartree-Fock Equations with Coulomb Two-Body Interaction

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Abstract. The existence and uniqueness of global solutions to the Cauchy problem is proved in the space of "smooth" density matrices for the time-dependent Hartree-Fock equations describing the motion of finite Fermi systems interacting via a Coulomb two-body potential.

1. Introduction

In this note, we indicate how to generalize the recent results of Bove, Da Prato, and Fano [1] concerning the time-dependent Hartree-Fock equations with bounded two-body interaction to include the Coulomb two-body interaction. (See this work and the references therein for a discussion of the origin of the problem.) Specifically we consider the existence of global solutions to the Cauchy problem for the equations

$$idK/dt=[\tfrac{1}{2}\Delta - U, K]_{-}\,, \tag{1.1}$$

where $K=K(t)$ is a density matrix [i.e. a non-negative trace class operator on $L^2(R^3)$] and U is the self-consistent potential $U_D - U_{\mathrm{EX}}$ defined by

$$(U_D f)(x)=(\int |x-y|^{-1}k(y,y;t)dy)f(x) \tag{1.2}$$

and

$$(U_{\mathrm{EX}} f)(x)=-\int |x-y|^{-1}k(x,y;t)f(y)\,dy \tag{1.3}$$

when $K(t)$ is represented as the integral operator $(K(t)f)(x)=\int k(x,y;t)f(y)dy$. The idea of the argument is to extend to this situation our results [2] for N-electron systems governed by the Hartree-Fock equations

$$i\,\partial\varphi_j/\partial t=\tfrac{1}{2}\Delta\varphi_j - U_{\mathrm{op}}\varphi_j\,, \tag{1.4}$$

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