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Quantized Fields in External Field

II. Existence Theorems

J. Bellissard*

Centre de Physique Théorique, C.N.R.S., F-13274 Marseille Cedex 2, France

Abstract. This is the second part of an article devoted to the study of quantized fields interacting with a smooth classical external field with fast space time decrease. The case of a charged scalar field is considered first. The existence of the corresponding Green's functions is proved. For weak fields, as well as pure electric or scalar external fields, the Bogoliubov S-operator defined in Part I of this work is shown to be unitary, covariant, causal up-to-a-phase. Its perturbation expansion is shown to converge on a dense set in Fock space. These results are generalised to a class of higher spin quantized fields, "nicely" coupled to external fields, which includes the Dirac theory, and, in the case of minimal and magnetic dipole coupling, the spin one Petiau-Duffin-Kemmer theory. It is not known whether this class contains examples of physical interest involving quantized fields carrying spins larger than one.

I. Introduction

In the first part [1] of this paper, we have described some general facts concerning the Green functions and the Bogoliubov S-operator for the problem of a spin zero quantized field interacting with an external field. The main result was that the S-operator constructed according to the perturbation scheme is covariant, unitary, and causal up-to-a-phase if and only if there exists a "non-perturbative" solution (N-P in [1], Definition II.2.3) of the following integral equation:

$$I = A + A\Delta_F I = A + I\Delta_F A \tag{I.1a}$$

with

$$A(x, y) = [v(x) + A_\mu A^\mu(x)]\delta(x-y) + i[A_\mu(x) + A_\mu(y)]\partial^\mu \delta(x-y)\,. \tag{I.1b}$$

v and A_μ denoting respectively the scalar and vector parts of the external field, and Δ_F the usual Feynman propagator.

Assuming the existence of a N-P solution the main result of [1] mentioned can easily be extended from the case of a spin zero quantized field to arbitrary integer spin. For half integer spin the algebraic properties of the Green functions undergo some well-known modifications [4a, 38] as explained in [1]. The method used in [1] to construct S have to be changed [2, 6a, 6b].

* Université de Provence et Centre de Physique Théorique, CNRS Marseille.