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Unbounded Derivations and Invariant Trace States

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Abstract. Let $\mathfrak{M}$ be a von Neumann algebra with cyclic trace vector Ω. Let $\delta(A)=i[H, A]$ be a spatial derivation of $\mathfrak{M}$ implemented by an operator H such that $H\Omega=0$ and H is essentially self-adjoint on $D(\delta)\Omega$.
It follows that:

$$e^{itH}\mathfrak{M}e^{-itH}=\mathfrak{M}, \quad t\in\mathbb{R}.$$

1. Introduction

In a previous paper [1] we discussed the general theory of unbounded derivations of a von Neumann algebra $\mathfrak{M}$ on a Hilbert space $\mathscr{H}$ and, in particular, introduced the notion of a spatial derivation. This latter form of derivation is defined in terms of a symmetric operator H, on $\mathscr{H}$, and a weakly dense *-subalgebra $D(\delta)$ of $\mathfrak{M}$, which leaves the domain $D(H)$ of H invariant. The derivation δ is defined to be a mapping

$$A\in D(\delta)\rightarrow\delta(A)\in\mathfrak{M}$$

with the property that

$$\delta(A)\psi=i[H, A]\psi, \quad \psi\in D(H).$$

It is of particular interest to study the case that H is self-adjoint and has an eigenvector Ω such that $D(\delta)\Omega$ is a core of H. In [1] it was conjectured that if Ω is also cyclic and separating for $\mathfrak{M}$ then

$$e^{itH}\mathfrak{M}e^{-itH}=\mathfrak{M}, \quad t\in\mathbb{R}.$$

This conjecture was verified in various special cases. If $\mathfrak{M}$ is abelian then it is essentially a theorem of Gallavotti and Pulvirenti [2]. In this note we extend the abelian result by verifying the conjecture whenever Ω is a trace vector.

2. Main Theorem

Theorem 1. *Let $\mathfrak{M}$ be a von Neumann algebra on a Hilbert space $\mathscr{H}$ and let Ω be a cyclic normalized vector defining a trace on $\mathfrak{M}$, i.e.*

$$(\Omega, AB\Omega)=(\Omega, BA\Omega), \quad A, B\in\mathfrak{M}.$$

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