Commun. math. Phys. 45, 279—294 (1975)

Communications in Mathematical Physics

Volume Dependence of Schwinger Functions in the Yukawa$_2$ Quantum Field Theory

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Abstract. We prove upper bounds on the partition function and Schwinger functions for the Euclidean Yukawa$_2$ quantum field theory which depend on the interaction volume Λ only through a term of the form (const)$^{|\Lambda|}$. We also prove a lower bound of the form (const)$^{|\Lambda|}$ for the partition function. We work throughout in the Matthews-Salam representation with the fermions integrated out.

I. Introduction

We study the Yukawa$_2$ quantum field theory in a finite volume Λ as a Euclidean boson field theory with the fermions "integrated out". The possibility of integrating out the fermions in the Yukawa theory was first demonstrated, in the external boson field case, by Matthews and Salam [1, 2], and in the finite volume interacting theory, by Seiler [3] who showed that the resulting Fredholm determinants are integrable functions of the boson field. As a step towards taking the infinite volume limit of Yukawa$_2$ we show in this paper that these determinants approximately factor over a decomposition of the space-time volume into subvolumes. While the determinants do not factor exactly, we exhibit upper and lower bounds which factor. The existence of such an approximate factoring is related to the exponential decoupling of distant regions in the free boson and fermion two point functions—i.e., to the nonzero free boson and fermion masses μ_0, m_0.

Our principal results are bounds on the un-normalized finite volume Schwinger functions

$$(ZS)^{(\Lambda)}(f_1,\ldots,f_n;g_1,\ldots,g_m;h_1,\ldots,h_m)$$
$$\equiv\langle\prod_{i=1}^n\phi(f_i)\prod_{j=1}^m\Psi^{(1)}(g_j)\prod_{k=1}^m\Psi^{(2)}(h_k)e^{-V(\Lambda)}\rangle\,,$$

and on the partition function $Z^{(\Lambda)}\equiv\langle e^{-V(\Lambda)}\rangle$. Here f_i, g_j, h_k are functions in the boson and fermion test-function spaces: $\mathscr{H}_{-1}^{(\mu_0)}$ and $\mathscr{H}^*=\mathscr{H}_{-\frac{1}{2}}^{(m_0)}\otimes C^2$, where $\mathscr{H}_s^{(m)}=L_2(R^2,(k^2+m^2)^s d^2k)$. We cover space-time with a lattice of unit squares Δ_α, with centers $\alpha\in Z^2$, and we suppose that the f_i are localized in unit squares.

* This work was supported in part by the National Science Foundation under Grant MPS 74-13252.