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Equilibrium Distributions of Physical Clusters

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Abstract. We consider classical systems of particles in $\mathbb{R}^d$ interacting by a stable pair potential with finite range. We are engaged in subdividing every particle configuration into clusters of interacting particles and studying the cluster distributions corresponding to equilibrium particle distributions.

Introduction

Let us consider an interaction in the d-dimensional Euclidean space $\mathbb{R}^d$ given by a pair potential Φ, i.e., the potential energy of particles located at $x_1, \ldots, x_n \in \mathbb{R}^d$ is given by

$$V(x_1, \ldots, x_n) = \sum_{1 \leqq i < j \leqq n} \Phi(x_j - x_i),$$

where $\Phi : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ is Lebesgue measurable with $\Phi(x) = \Phi(-x)$ for $x \in \mathbb{R}^d$. We suppose Φ to have the following properties:

stability: there exists $B \geqq 0$ with $V(x_1, \ldots, x_n) \geqq -nB$ for all n and $x_1, \ldots, x_n \in \mathbb{R}^d$;

finite range: there exists $R > 0$ with $\Phi(x) = 0$ for $|x| > R$.

Because of the finite range property it is reasonable to introduce clusters of interacting particles. Thus a configuration $(x_1, \ldots, x_l)$ is a cluster, iff each two particles of the cluster interact at least indirectly.

This is a special type of physical clusters introduced in 1939 independently by Frenkel and Band in order to discuss condensation phenomena (see [3]). Recently Sinai [8] defined similar clusters – clusters in space-time however – for the existence of the time evolution of particle configurations.

Every finite or infinite particle configuration can now be subdivided into clusters with possibly infinite clusters defined in the same way. The purpose of this paper is to study the distribution of cluster configurations corresponding to equilibrium particle distributions in the sense of the DLR-equations in the case of only finite clusters with probability 1.

In Section 1 we give the exact definition of clusters by means of cluster functions and denote relations of these functions describing the subdivision of finite particle configurations into clusters. These relations are used in Section 2 to derive the cluster distribution corresponding to a grand canonical particle