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Asymptotic Analysis of Deterministic and Stochastic Equations with Rapidly Varying Components

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Abstract. The asymptotic character of deterministic and stochastic equations whose solutions have a rapidly varying component is studied. Of particular interest is the class of problems for which the limiting behavior can be described in a contracted and simplified framework.

1. Introduction

The object of this paper is the asymptotic analysis of equations whose solutions have the following three features: (i) rapidly oscillating components, (ii) rapidly decaying components, (iii) rapidly varying stochastic components. The asymptotic limit that emerges can be described in a considerably smaller space and is much simpler than the original problem. This contraction of the description due to the three features above is what we seek to analyze.

Naturally, problems of this form have received considerable attention due to their frequent appearance in many different areas of physics and elsewhere. Many of our references contain in one form or another such problems. In [1] and [2] the passage from the linearized Boltzmann equation to the linearized Navier-Stokes equation is considered. This problem is reconsidered here in Section 2. Our analysis is similar to that of [3] and [4] and we employ it because it extends easily to stochastic problems. We refer also to the work on Generalized Master equations [5] and in particular to its mathematical development [6] which is very similar to our problem. The work of Kurtz [7] also aims in the same general direction as does the work on random evolutions [8, and references therein].

In Section 3 we consider the asymptotic behavior of problems in the form of stochastic Boltzmann equations. Such problems are of interest in the study of transport phenomena in random media [9] and elsewhere. In Section 4, we consider

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