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Phase Transitions for φ_2^4 Quantum Fields*

James Glimm**
Rockefeller University, New York, N.Y. 10021, USA

Arthur Jaffe*** and Thomas Spencer***, †
Harvard University, Cambridge, Mass. 02138, USA

Abstract. Phase transitions for the quantum field interaction $\lambda\varphi^4 + m_0^2\varphi^2$, $m_0^2/\lambda \ll 1$ are established in two dimensional space time.

1. Introduction

We present a direct proof of the existence of phase transitions in quantum field theory. We consider here the simplest interaction for which a phase transition is expected, namely the

$$\lambda\varphi^4 + \tfrac{1}{2} m_0^2\varphi^2, \qquad m_0^2/\lambda \ll 1, \tag{1.1}$$

perturbation of the free field of mass m_0. We give a complete proof in space time dimension $d=2$. Our same methods apply in principle to arbitrary even $P(\varphi)_2$ models without cutoff.

To define the interaction (1.1) for $d=2$ we require Wick ordering. We denote Wick ordering of P with respect to the covariance $(-\Delta + m_0^2)^{-1}$ by $:P:_{m_0}$. Then scaling and re Wick ordering leads to an equivalent theory with the bare mass $O(\sigma)^{-1}$ and the interaction which we study, see [13], is

$$:P(\varphi):_{\sigma^{-1}} = :(\varphi^2 - \sigma^2)^2/\sigma^2:_{\sigma^{-1}}, \qquad \sigma \gg 1. \tag{1.2}$$

It is the occurrence of two distinct minima, separated by a large barrier, which suggests the occurrence of phase transitions for the interaction (1.2). The two pure phases are ground states localized (in φ space) near the two minima $\varphi = \pm\sigma$.

In the case we consider, the polynominal $P(\varphi)$ is invariant under the symmetry transformation $\varphi \to -\varphi$, while the pure phases are interchanged by the symmetry. We note, however, that symmetry breaking is a distinct issue from the existence of phase transitions. Just as in statistical mechanics, where phase transitions may occur without symmetry breaking [11], we expect phase transitions in field theory for certain $P(\varphi)$ models which do not possess a symmetry group, such as

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† On leave from Rockefeller University, New York, NY 10021, USA.