Commun. math. Phys. 45, 137—151 (1975)

Communications in Mathematical Physics

Prime Field Decompositions and Infinitely Divisible States on Borchers' Tensor Algebra

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Abstract. We generalize some notions of probability theory and theory of group representations to field theory and to states on the Borchers algebra $\underline{\mathscr{S}}$. It is shown that every field (relativistic and Euclidean, ...) can be decomposed into a countable number of prime fields and an infinitely divisible field. In terms of states this means that every state on $\underline{\mathscr{S}}$ is a product of an infinitely divisible state and a countable number of prime states, and in this formulation it applies equally well to correlation functions of statistical mechanics and to moments of linear stochastic processes over $\underline{\mathscr{S}}$ or $\underline{\mathscr{D}}$. Necessary and sufficient conditions for infinitely divisible states are given. It is shown that the fields of the ϕ_2^4-theory are either prime or contain prime factors. Our results reduce the classification problem of Wightman and Euclidean fields to that of prime fields and infinitely divisible fields. It is pointed out that prime fields are relevant for a nontrivial scattering theory.

1. Introduction and Main Theorem

The motivation of this paper comes from quantum field theory although the results have a wider range of applications. Let us consider, for example, two relativistic scalar fields $\varphi_1(x)$ and $\varphi_2(x)$ in Hilbert spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$, respectively, satisfying all Wightman axioms, including uniqueness of the vacua Ω_1 and Ω_2. We can then form the field

$$\varphi(x) := \varphi_1(x) \times \mathbb{1}_2 + \mathbb{1}_1 \times \varphi_2(x) \tag{1.1}$$

and consider it in the subspace $\mathfrak{H}$ of $\mathfrak{H}_1 \times \mathfrak{H}_2$ in which $\Omega := \Omega_1 \times \Omega_2$ is cyclic. Then φ again satisfies all Wightman axioms and is irreducible in $\mathfrak{H}$. Equation (1.1) is analogous to the Kronecker product of Lie algebras, and we write

$$\varphi = \varphi_1 \, s \, \varphi_2 \,. \tag{1.2}$$

It is natural now to ask the reverse question: Can a given Wightman field φ be written as an s-product of other Wightman fields? Clearly a similar problem can be formulated for other classes of fields.

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