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The Free Boson Gas in a Rotating Bucket

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§ 1. Introduction

Putterman, Kac and Uhlenbeck [4] have proposed a purely quantum mechanical explanation for the origin of vortex lines in rotating He II, suggested by considering an ideal Boson gas in a rotating cylindrical bucket. Blatt and Butler [3] showed that a rotating ideal Boson gas undergoes phase transitions similar to those occurring in rotating He II. Their main result is that the total angular momentum J of the gas, considered as a function of the angular velocity ω of the bucket, increases linearly between a sequence $\omega_1, \omega_2, \ldots$ of critical values of ω. At a critical value of ω the angular momentum jumps by an amount $N_0\hbar$, where N_0 is the number of condensed particles. For ω between two critical values ω_l and ω_{l+1} we have

$$J=\tfrac{1}{2}(N-N_0)mR^2\omega+N_0 l\hbar\,,$$

where N is the total number of particles, m is the mass of a particle and R is the radius of the bucket. The contribution from the $N-N_0$ particles in the normal fluid is as if they were in solid body rotation. The contribution from the N_0 particles in the condensate is what one expects from a quantized vortex line of strength $l\hbar/m$ on the axis of the bucket.

The centrifugal density distortion in the normal fluid is negligible, since ω is assumed to be of order $\hbar/mR^2$. Then the kinetic energy of rotation when m is the mass of a helium atom and R is 1 cm. is much smaller than kT for temperatures around 1 °K. This is not so for the condensate and these particles are pushed out to the rim of the contained for $\omega>\omega_1$. The reason for this behaviour can be seen informally as follows: The Bose condensation takes place into the single-particle state which is the ground state of the Hamiltonian $H=H_0-\omega J_3$ where H_0 is the free-particle Hamiltonian and J_3 is the operator corresponding to the component of angular momentum along the axis of the bucket. The eigenvalues of H are $E_k-\hbar\omega l$ where $k=(n,l,m)$ stands for the three quantum numbers appropriate to the cylindrical geometry and the E_k are the eigenvalues of H_0. We assume that the walls of the container are perfectly elastic so that the normal derivative of the wave-function is zero at the boundary. This is a reasonable assumption in thermal equilibrium (see [9]) but quite inappropriate for a discussion of the approach to equilibrium which would be necessary for an investigation of the formation of vortices (see [4]). The eigenfunctions of H are the same as those