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Bounds in the Yukawa$_2$ Quantum Field Theory: Upper Bound on the Pressure, Hamiltonian Bound and Linear Lower Bound*

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Abstract. We prove bounds of the form $Z_\Lambda \leqq e^{a|\Lambda|}$ and $(\mathrm{SZ})_\Lambda \leqq e^{a|\Lambda|}$ in the Y_2 Euclidean field theory and from this obtain Glimm's Hamiltonian bound and Schrader's linear lower bound.

I. Introduction

One of the most basic infinite volume bounds in constructive field theory is a linear lower bound on the energy per unit volume. Such bounds were first proven in $P(\phi)_2$ theories by Glimm and Jaffe [6], Y_2 theories by Schrader [18] and ϕ_3^4 theories by Glimm and Jaffe [7]. The Euclidean translation of these bounds fits into the view of Euclidean field theories as statistical mechanical systems [10, 8] for the "essentially equivalent" Euclidean bound is an upper bound on the pressure (see [10], § VI).

Our goal in this paper is to provide a new proof of and, we feel, new insight into Schrader's bound. Along the way we will establish Glimm's basic result [4, 5] that the (renormalized) Yukawa$_2$ Hamiltonian spatially cutoff is bounded from below. (This result is a basic input in Schrader's proof.) We also prove a volume independent bound on the Euclidean pressure – in fact this is our main input in proving the results of Glimm and Schrader. Conversely, we should note that given the connection between the Hamiltonian and Euclidean theories (see [15], § III for this connection, which uses the Euclidean Fermi fields of Osterwalder-Schrader), Schrader's result implies a bound on the pressure.

A "semi-Euclidean" proof of Schrader's result has been obtained by Brydges [1]. When our own work on this subject was completed in a preliminary draft, we received a preprint from McBryan [15] with similar results. McBryan also works

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