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A Formalism for the Investigation of Algebraically Special Metrics. II

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Abstract. The equations of the formalism developed in part I are simplified by specialisation of the basic pair of null directions. The auxiliary vectors previously introduced are shown to have intrinsic geometric properties which are directly related to the complexity of the differential operators of the formalism. An Ansatz based on these properties is introduced and the calculation of the metric so defined is used to display the connection between the coordinate-free and coordinate-dependent solutions.

Introduction

In this first paper [1] of this series a new technique for the investigation of empty space algebraically special solutions of the Einstein equations was introduced. In this work we develop the method further. We show that certain quantities which appear naturally in the course of calculation have a deeper significance and contain information about the intrinsic geometric structure of the space. We also demonstrate the efficacy of the method, showing how elementary arguments lead to simplifying Ansätze and how easily calculations may be done.

The first part of the paper contains a review of the method and a compact statement of the equations governing this type of metric. We show that for a particular choice of tetrad these equations may be partially integrated in a coordinate free sense and the problem thereby reduced to a set of eight linear equations, all of which are extremely simple in form.

The problem of simplifying specializations suggested by the previously derived equations are considered. This leads us to investigate the auxilliary vectors $\tilde{\alpha}$ and $\tilde{\beta}$ which were introduced in Ref. [1]. These vectors contain the information inherent in the Newman-Penrose (N.P.) spin coefficients [2] α, β, γ, and ε, and are an essential part of the differential operators of the formalism. They are not of good weight [3] transforming inhomogeneously under the gauge transformations (cf. below), but despite this they contain important information concerning the intrinsic structure of the space and the solubility of the resulting equations is highly dependent on their properties.

The next part of the paper is devoted to a discussion of some properties common to all algebraically special empty-space metrics with special reference to a family of two surface metrics inherent in such solutions. Finally we derive the metric determined by a special Ansatz.