Coherent States and Symmetric Spaces

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Abstract. Properties of system of the coherent states related to representations of the class I of principal series of the motion groups of symmetric spaces of rank 1 have been studied. It has been proved that such states are given by horospherical kernels and are the generalization of the plane waves for the case of symmetric spaces.

1. Introduction

This paper deals with the further study of the system of generalized coherent states (CS). According to [1] the system of generalized CS of the type $(T, |\psi_0\rangle)$ is called the set of states $\{T(g)|\psi_0\rangle\}$ where $T(g)$ is the unitary irreducible representation (UIR) of the group G acting in the Hilbert space $\mathcal{H}$, g runs over all the group G and $|\psi_0\rangle$ is the fixed vector of the space $\mathcal{H}$. Such states are a generalization of usual CS [2, 3] obtained in choosing the simplest nilpotent group (Heisenberg-Weyl group) as the group G and the so called vacuum vector as $|\psi_0\rangle$. Generalized CS, as well as usual CS, turn out to be very convenient for the solution of a number of physical problems possessing dynamical symmetry.

Thus, for instance, in papers [4] the problems of boson and fermion pair creation in alternating homogeneous field were solved with their help. In paper [5] CS for rotation group (previously introduced in paper [6]) were used to obtain estimates for the partition function of a quantum spin system. In papers [7, 8] such states were applied in the so called Dicke model describing the interaction of radiation with the matter.

In the following we shall call generalized CS for brevity simply CS. Note that the CS system is an overcomplete and nonorthogonal system of vectors (states) of Hilbert space. Under the additional assumption on square integrability a number of properties of such systems was considered in papers [9–13].

In this paper we study some CS systems which are not square integrable. namely the systems related to UIRs of the class I of the principal series of symmetric space motion groups. We recall that the representation of class I of the real semisimple Lie group is called the representation $T(g)$ in whose representation space there exists a vector $|\psi_0\rangle$ invariant relative to action of the maximal compact subgroup K of this group. Let us consider the CS system of the type $(T, |\psi_0\rangle)$. It is easy to see that the coherent state of this type is determined by the point of symmetric space $X = G/K$ [1]. In the case of the so called principal series

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[1] In this paper we restrict ourselves by consideration of the case of symmetric spaces of rank I. Note that for the SO(3, 1) group (the Lorenz group) such system of states coincides with that introduced from another considerations in a paper by Shapiro [14]. This system proved to be very convenient for considering a number of problems of the Lorenz group representation theory. For the case of the SO(n, 1) group a number of useful formulae related to the questions under consideration may be found in book [15].