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Spontaneous Breakdown in Two Dimensional Space-Time

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Abstract. We prove that in two-dimensional space-time, symmetry transformations which are generated by Poincaré covariant currents can not be spontaneously broken. This is also the case with the dilation current. We argue that other currents which involve explicit space-time dependence might lead to spontaneously broken symmetries accompanied by massless Goldstone bosons. We construct a trivial example where this phenomenon occurs.

Introduction and Statement of the Results

In Ref. [1] it was shown that in two dimensional space-time, the following result holds: Let $\phi(x)$ be a scalar field and $j^\mu(x)$ a conserved vector field, then

$$\int dx^1(\Omega, [j^0(x), \phi(y)]\Omega)=0 \tag{1}$$

where Ω is the vacuum state. This is an indication that in two dimensional space-time, a symmetry transformation which is generated by $j^\mu(x)$ is always an exact one and cannot be spontaneously broken. There still exists the question what happens if one takes in Eq. (1) instead of $\phi(y)$ an arbitrary polynomial in tensorial and spinorial fields, or if one considers currents with different transformation properties under the Poincaré group. We discuss these questions here.

The framework is Wightman field theory [2] in two dimensional space-time. Let A denote any polynomial in the smeared fields where the smearing functions are infinitely differentiable and of compact support. Let $j^\mu(x)$ denote a set of four fields satisfying a conservation equation

$$\partial_\mu j^\mu(x)=0\,. \tag{2}$$

The Goldstone theorem states that [3]

$$\int dx^1(\Omega, [j^0(x), A]\Omega)=\int dx^1(\Omega, [j^0(x)E_0A-AE_0j^0(x)]\Omega) \tag{3}$$

where E_0 is the projection on the subspace of zero mass states. In Eq. (3), $j^\mu(x)$ does not have to be a vector field, actually, it can even contain an explicit space-time dependence through polynomials in x^μ (see Ref. [4]). For instance, one can take the dilation current

$$d^\mu(x)=x_\nu T^{\mu\nu}(x) \tag{4}$$

or the special conformal currents

$$c^{\mu\nu}(x)=(2x^\nu x_\lambda-\delta^\nu_\lambda x^2)T^{\mu\lambda}(x) \tag{5}$$