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Inner*-Automorphisms of Simple C^*-Algebras

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Introduction

Given a locally compact abelian group G acting as *-automorphisms α_g on a factor, Connes ([3]) defines a certain subgroup $\Gamma(\alpha)$ of the dual group Γ of G. He shows that under suitable conditions the annihilator of $\Gamma(\alpha)$ is precisely the subgroup of $h \in G$ for which the automorphism α_h is implemented by a unitary element in the centre of the fixed-point algebra of the group. As a corollary it is proved that if a single *-automorphism α has a spectrum (as a bounded operator on the factor) which is not the entire unit circle, then a power of α is inner.

In [2], Borchers proves that on any von Neumann algebra a *-automorphism with a gap in its spectrum has a power which is inner.

Here we generalize the notion of $\Gamma(\alpha)$ to representations of G as *-automorphisms acting on an arbitrary C^*-algebra. We show in 2 that $\Gamma(\alpha)$ is a closed subgroup of Γ, which satisfies $\Gamma(\alpha)+\mathrm{sp}\alpha \subseteqq \mathrm{sp}\alpha$.

In Section 3 we see that for primitive C^*-algebras, the spectra of restricted actions α^B on non-zero α-invariant hereditary C^*-subalgebras B form an approximately filtering family of sets (in a sense made precise in 3.4). The methods of [3] are then applicable to simple C^*-algebras, and we show in 4 that for suitable groups the annihilator of $\Gamma(\alpha)$ is precisely the subgroup of $h \in G$ for which α_h is implemented by a unitary element in the centre of the fixed-point algebra of the bitransposed action on the multiplier algebra. From this it follows that a single *-automorphism with a gap in its spectrum has a power which is given by a multiplier.

Studying a single *-automorphism α on a C^*-algebra we show in Section 5 that the methods of [2] may be generalized to give the result that if $\sigma(\alpha)$ has a gap, then some power α^n is the exponential of a derivation on a non-zero α-invariant hereditary C^*-subalgebra.

When A is a commutative C^*-algebra, this method of proof yields that α^n for a suitable n is the identity operator on A. In fact, it is noted that with slight modifications the arguments given carry over to the case where α is an isometric isomorphism of a commutative semi-simple Banach algebra. This result has earlier been proved in [6] and [7], using different methods.

In Section 6, we return to the group setting in the special case where A is a von Neumann algebra, and reach some generalizations of results obtained for factors in [3]. We also obtain the result in [2] that a *-automorphism with gap in its spectrum has a power which is inner.