Commun. math. Phys. 44, 169—173 (1975)

A Remark to Harris's Theorem on Percolation

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Received March 25, 1975

Abstract. Harris's theorem on percolation is generalized to a dependent case by his own method.

Let T be the set of bonds in the plane square lattice Z^2. We adopt the following

Notations.
$\Omega = \{0, 1\}^T$; the set of configurations of 0 and 1 in T.
$X_t(\omega) = \omega(t)$ for $t \in T$ and $\omega \in \Omega$.
$\boldsymbol{X} = \{X_t; t \in T\}$.
$\boldsymbol{X}^{-1}(i) \equiv \boldsymbol{X}^{-1}(i, \omega) = \{t \in T; X_t(\omega) = i\}$ for $i = 0, 1$.
P is a probability measure on Ω.

Harris [3] proved following

Theorem. *If a random field $(\Omega, P; \boldsymbol{X})$ is independent at each $t \in T$, and if $P(X_t = 0) = P(X_t = 1) = 1/2$ for every t, then neither $\boldsymbol{X}^{-1}(0)$ nor $\boldsymbol{X}^{-1}(1)$ has infinite connected components a.s.*

His method can be applied to generalize the above to a dependent case. For $V \subset T$, let $\mathscr{B}_V$ be the σ-algebra generated by $\{X_t; t \in V\}$, and let $\mathscr{B}_\infty = \bigcap_V \mathscr{B}_{V^c}$, where V runs over the set of all finite subsets of T. Let ∂V be the set of bonds which meet bonds in V at right angles.

Theorem. *We assume that a random field $(\Omega, P; \boldsymbol{X})$ satisfies the following conditions;*

(1) *(Spatial symmetry) P is invariant under shift, rotation by right angles and reflection in the axis in T.*

(2) *(Symmetry of configurations) P is invariant under interchange of* 0 *and* 1.

(3) *P is everywhere dense.*

(4) *$\mathscr{B}_\infty$ is trivial, if it is measured by P.*

(5) *(The* FKG *inequality) If $f(\omega)$ and $g(\omega)$ are non-decreasing functions of $\omega \in \Omega$, then*

$$\int_\Omega f(\omega) g(\omega) P(d\omega) \geqq \int_\Omega f(\omega) P(d\omega) \cdot \int_\Omega g(\omega) P(d\omega) .$$

(6) *(Markovian property) For each $A \in \mathscr{B}_V$,*

$$P(A | \mathscr{B}_{V^c}) = P(A | \mathscr{B}_{\partial V}) .$$

Then, neither $\boldsymbol{X}^{-1}(0)$ nor $\boldsymbol{X}^{-1}(1)$ has infinite connected components a.s.