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Particle Lattice Models and the Dynamical Instability of Many-Body Systems*

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Abstract. We consider the notion of dynamical instability of many-body systems wherein states, which are arbitrarily close initially, are not close at some other fixed time. In controlling the dynamics of interacting systems of identical Fermions moving on a lattice, we isolate a basic mechanism which causes instability.

1. Introduction

There are few calculations which establish qualitative control over the dynamics of interacting, many-body, nonrelativistic systems *as a function of particle number*. We are particularly interested here in controlling the following "instability" phenomenon: A many-body system, which we assume to be qualitatively independent of its (large but finite) particle number n and volume V_n, can admit a naturally related family of initial states f_n with the property that for each n there is a well defined evolution of f_n for all time, and yet for which there is an *instability in the finite particle system* which is made manifest when the initial state $f_\infty = \lim_{n\to\infty} f_n$ only has a well defined evolution for finite time. For example, one can easily set up a classical mechanical system of n point particles, $p^1, \ldots, p^n$, and an initial state f_n such that at time $t = t_j = \sum_{k=1}^{j} 2^{-k}$ particle p^j hits the "target particle" p^1 imparting a unit of momentum in a fixed direction. It is clear that for large but finite n, something unusual occurs just before $t=1$ (p^1 attains arbitrarily high momentum) which causes the breakdown of the evolution of f_∞ at $t=1$. We emphasize that the "catastrophic" feature of the infinite particle system is only a manifestation of a real instability of the n-particle system and not just an anomaly of infinite particle systems. (We use the term "instability" because even though f_n and f_m are arbitrarily close, being close to f_∞, their evolved states $f_n(t)$ and $f_m(t)$ cannot be close at $t=1$ or else they would define a state, $f_\infty(1)$, which we know does not exist.) A parallel with the phenomenon of phase transitions is clear — models of finite particle systems do not exhibit phase transitions in the sense of actual discontinuity or nondifferentiability of thermodynamic functions of temperature, but the inherent instability of the finite particle system is made manifest by these features of the corresponding infinite particle system.

Instability seems to be a key obstacle in the dynamical control of continuous quantum mechanical nonrelativistic many-body systems, though we are unaware of any convincing proof that instabilities or catastrophies, of the type described above, can actually occur in quantum systems; see [1—4]. The main goal of this

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