

# Comments on Lightlike Translations and Applications in Relativistic Quantum Field Theory

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**Abstract.** In the algebraic framework of quantum field theory we consider one parameter subgroups of lightlike translations. After establishing a few preliminary properties we prove a certain cluster property and then exhibit the close connection between such subgroups and a class of type III factors. A few applications of this connection are also discussed.

## Introduction

When looking for applications of spectrum condition together with locality in relativistic QFT one finds that up to this time the possibilities of using lightlike translation subgroups of the Poincaré group in connection with geometric properties of Minkowski space have not yet been fully realized.

In the present work we shall argue that, roughly speaking, “thinking in lightlike terms” may be useful not only in elementary particle physics but also in general relativistic QFT leading to

- a) new proofs and strengthened versions of existing results,
- b) entirely new results which would be specific for relativistic theories and which could probably not be obtained otherwise.

In Section 1 we formulate our assumptions and state a few preliminary results on lightlike translations in field theory, some of which will not be used in the sequel of this work but might be useful in further studies.

In Section 2 we prove the main theorem, i.e. by use of the properties of lightlike translations we show that the von Neumann algebras belonging to certain unbounded regions are always type III factors.

The applications (discussed in Section 3) are not very strong by themselves and should be judged more from the methodological point of view; we hope, however, that they will be sufficient to convince the reader that the role of lightlike translations in relativistic field theory deserves more thorough investigation.

## 1. Lightlike Translations in Relativistic Field Theory

We shall work in the algebraic framework of QFT and assume:

α)  $\mathfrak{A}$  is a  $C^*$ -algebra of quasilocal operators in a separable Hilbert space  $\mathcal{H}$  and is generated in the usual way by local von Neumann algebras  $\mathfrak{R}(\mathcal{O})$  which belong to bounded open regions (b.o.r.) in Minkowski space  $\mathbb{R}^n (n \geq 2)$  and are subject to locality and isotony.