Commun. math. Phys. 44, 107—124 (1975)

On the Free Boson Gas with Spin

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Received March 25, 1975

Abstract. The generating functionals of the grand canonical and canonical thermodynamic equilibrium states of several models of free bosons with spin are calculated and the properties of the states discussed. In particular the distribution of condensate over the degenerate ground state is described, and it is shown that spinning bosons interacting with a magnetic field exhibit spontaneous magnetization at sufficiently low temperatures.

§ 1. Introduction and Summary of Results

Cannon [1], Lewis and Pulè [2] have rigorously studied the thermodynamic limit of the canonical and grand canonical equilibrium states of the free boson gas at arbitrary density and temperature. We extend their methods to three models in which the free bosons have spin v. The aim is to prove the existence of Bose-Einstein condensation, and to describe its nature. In particular we look at the fluctuations in the density of condensate, comparing the canonical and grand canonical ensembles, and we investigate a suggestion of Buckingham [4] and Blatt [3] that a condensed gas of spinning bosons with a magnetic moment would behave like a ferromagnet.

In § 2 we discuss the basic model of free spinning bosons in both the grand canonical ensemble (g.c.e.) and the canonical ensemble (c.e.). It is similar to the case $v=0$. There is a critical density ϱ_c, and for densities greater than this critical value Bose-Einstein condensation occurs, the condensate being equally distributed over the $2v+1$ zero energy states. In both ensembles the density of condensate in each of the condensed states fluctuates (more so in the g.c.e. than in the c.e.), but only in the canonical ensemble do these fluctuations cancel out, so that the total condensate density does not fluctuate [see Eqs. (2.13)–(2.17)].

The spin plays a very subsidiary role in this model. Essentially it provides a means of achieving a degenerate ground state. We could have considered a non-interacting system of $2v+1$ different species of particles with identical masses (and so identical energy spectra) and arrived at the same answer; or, with only a slight modification, we could have treated the case where the different species have different masses. Qualitatively the result would be unchanged.

In the model considered in § 3 the spin plays a more significant role; the various spin states (or, equivalently, the various species) have different properties. In this model, in addition to demanding that the mean particle density is fixed at $\bar{\varrho}$, we also require the mean spin density to take a prescribed value $\bar{s}$. The condensation is more complicated than in the model of § 2 and its nature depends on the value