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Automorphisms and Quasi-Free States of the CAR Algebra

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Abstract. We study automorphisms of the CAR algebra which map the family of gauge-invariant, quasi-free states of the CAR algebra onto itself and show (Theorem 3.1) that they are one-particle automorphisms.

1. Introduction

The problem discussed in this paper arose from questions regarding equilibrium states of thermodynamical systems. Equilibrium states have been extensively discussed in the framework of C^*-algebras of observables (see, for example [6, 15]). Such states are labeled by a very small number of parameters. For example, in the case of a gas of identical particles, the equilibrium states are labeled by the temperature, the chemical potential, and the average velocity of the particles – quantities which relate directly with the conserved quantities – energy, particle number, and total momentum. Since conserved quantities are in one-one correspondence with one-parameter groups of transformations which leave the Hamiltonian invariant, we can describe the situation in a way which remains meaningful for infinite systems. Equilibrium states of thermodynamical systems are labeled by a very small number of parameters which relate directly with one-parameter, automorphism groups of the observable algebra that commute with the time-evolution automorphisms. The fact that there are so few parameters involved, which is related to the fact that there are only a small number of one-parameter groups of automorphisms that commute with the time-evolution, is an aspect of the ergodic nature of most large physical systems. A proof based on the dynamics of the system is still lacking, even though Sinai has obtained very interesting results in this direction for a classical system of N hard spheres.

Systems of particles without interaction do not behave ergodically in the above sense. This does not mean, however, that systems of noninteracting particles are uninteresting from the point of view of ergodicity. In [7, 8], the asymptotic time behavior of the free Fermi gas is discussed. It is found in [7] that, for increasing time, primary states of the CAR algebra are asymptotic to gauge-invariant, quasi-free states, provided these states satisfy a certain clustering property. In particular, primary, stationary (i.e. time-invariant) states with that clustering property are quasi-free. Quasi-free states [1–5, 9–14, 16, 17] are particularly simple states in the sense that they lack all except two-point correlations. Some