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An Existence Theorem for a Massive Vector Meson in an External Electromagnetic Field

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Abstract. The system of Lagrangian equations describing a spin one particle moving in an external electromagnetic field with minimal, dipole and quadrupole interactions is shown to be equivalent to a symmetric hyperbolic system of partial differential equations, to which a standard existence theorem can be applied. The key hypothesis of the treatment is that the derivatives of the electromagnetic field must be sufficiently small. The results cover also the case of noncausal propagation of signals.

1. Introduction

In a preceding paper [1] (from now on referred to as I) an existence theorem for the system of equations describing a massive vector meson interacting with an external tensor field was proved. The result was obtained by essentially reducing the system of Lagrangian equations to a symmetric hyperbolic system of partial differential equations (PDE) to which standard existence theorem could be applied.

Here we propose to establish an analogous existence theorem for a massive spin one particle in a suitable external electromagnetic field with minimal, dipole and quadrupole couplings. The result is known for minimal interactions [2] and for minimal and dipole interactions [3]. The novelty resides in the treatment of the quadrupole coupling, which is a much less regular case because there can be solutions propagating at a speed greater than the speed of light, that is outside the light cone [4].

The Lagrangian we will start from is essentially that of Bludman and Young [5], but to establish a connection with the Lagrangian used in Ref. [4], we have to add an extra interaction term containing a symmetric external tensor field. In this sense this paper is not only a continuation of I, but it embodies most of the results of I.

The ideas developed here are the same as in I to which we refer both for a description of the situation concerning in general existence theorems and for details about the type of treatment here employed.

In Section 2. we will establish an equivalence theorem between the initial Lagrangian equations and the new equations we will consider. Section 3. contains the proof that, for suitably small external fields, the new system is equivalent to a symmetric hyperbolic system, leading in this way to the existence theorem. A computation of a characteristic determinant is performed in the Appendix.