Commun. math. Phys. 43, 155—159 (1975)

A Family of Codes between Some Markov and Bernoulli Schemes

Gabriella Monroy and Lucio Russo
Istituto di Fisica Teorica, Università di Napoli, Napoli, Italy

Received February 13, 1975

Abstract. We construct a family of almost continuous codes between a mixing one-step Markov process with two symbols and a Bernoulli scheme.

1. Introduction

It is known (see Ref. [2]) that two "finitely determined" processes with the same entropy are isomorphic. In particular this statement implies that a mixing Markov chain is isomorphic to a Bernoulli scheme with the same entropy ([1, 2]). The isomorphism means, in the language of codings, that one can find invertible and shift-invariant maps between the typical sequences of the two processes. Such "codes", nevertheless, are explicitely known only in a few cases (see [4]); moreover, it is not known whether, endowing the space of sequences with the natural topology defined below, it is possible to construct almost continuous codes.

Here we construct explicitely an uncountable family of almost continuous invertible codes between a mixing one-step Markov process with two symbols and a Bernoulli scheme.

2. Symbols and Definitions

Let $X=\{-1,1\}^{\mathbb{Z}}$ be the set of all doubly infinite sequences of the two numbers $-1, 1$, $\mathscr{B}$ the σ-algebra of the Borel sets in X (endowed with the topology obtained as product of the discrete topologies on the factors); let $T: X \to X$ be the map defined by:

$$(Tx)_i = x_{i+1}\,. \tag{2.1}$$

If μ_S is a T-invariant measure defined on $\mathscr{B}$, we call the triple $S=(X, T, \mu_S)$ a "shift". If, moreover, $\varphi: X \to X$ is a Borel map commuting with T, let us define:

$$\bar{\varphi} S = (X, T, \bar{\varphi}\mu_S)\,, \quad \text{with} \quad \bar{\varphi}\mu_S(Y) = \mu_S(\varphi^{-1} Y) \qquad \forall Y \in \mathscr{B}\,.$$

Let $0<\alpha<1$; call B_α the (Bernoulli) shift whose measure μ_{B_α}, extended to $\mathscr{B}$ by Kolmogorov's theorem, is defined on the cylinders of X by:

$$\mu_{B_\alpha}(\{x \in X \mid x_{i+1} = k_1 \dots x_{i+l} = k_l\}) = \prod_{\substack{1 \le r \le l \\ k_r = 1}} \alpha \prod_{\substack{1 \le r \le l \\ k_r = -1}} (1-\alpha)\,. \tag{2.2}$$