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Representations of the CAR Generated by Representations of the CCR. Fock Case

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Abstract. We present a method of constructing the Fock representation of the canonical anticommutation relations in the Fock representation of the canonical commutation relations. An explicit formula for Fermi creation and annihilation operators in terms of Bose ones is given.

1. Introduction

The spinor theory of Heisenberg [1] is an example of the philosophy that a fundamental theory of elementary particles must involve Fermi rather than Bose fields in the basic formalism. Quite the contrary, there were many attempts [2, 3] and references there-in to describe fermions in terms of bosons. Streater and Wilde [2] have shown that in two-dimensional space-time fermion states of a boson field do exist.

Kalnay, MacCotrina and Kademova [3] succeeded to show that in the case of Fock representations, free Fermi field can be expanded into a sum of pairs of Bose operators.

We want to present an independent investigation showing that certain homomorphisms of the n-th power space $K^{\otimes n}$ can be utilized to produce the Fock representations of the CAR algebra expressed by infinite series in Bose creation and annihilation operators.

2. Notations

Let $\mathscr{K}$, $\mathscr{H}$ be complex Hilbert spaces with an involution $*, \mathscr{K} \ni f, g$. Let $\mathscr{U}_F(\mathscr{K})$ be a $*$ representation of the CAR algebra over $\mathscr{K}$ acting in $\mathscr{H} = \mathscr{H}_F$. The generating elements $b(f), b(g)^*$ fulfill:

$$\begin{aligned} &[b(f), b(g)^*]_+ = (f^*, g)\, \mathbb{1}_F \\ &[b(f), b(g)]_+ = 0 \end{aligned} \tag{2.1}$$

where $(\cdot, \cdot)$ is a bilinear form in $\mathscr{K}$, $\mathbb{1}$ is the unit operator in $\mathscr{U}_F(\mathscr{K})$. Let further $\mathscr{U}_B(\mathscr{K})$ be a $*$ representation of the CCR algebra over $\mathscr{K}$ acting in some $\mathscr{D} \subset \mathscr{H} = \mathscr{H}_B$, $\mathbb{1}_B$ is a unit operator in $\mathscr{U}_B(\mathscr{K})$. For generating elements $a(f), a(g)^*$ we have

$$\begin{aligned} &[a(f), a(g)^*]_- = (f^*, g)\, \mathbb{1}_B \\ &[a(f), a(g)]_- = 0\,. \end{aligned} \tag{2.2}$$