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Scattering Theory of the Linear Boltzmannoperator

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Abstract. In time dependent scattering theory we know three important examples: the wave equation around an obstacle, the Schrödinger and the Dirac equation with a scattering potential. In this paper another example from time dependent linear transport theory is added and considered in full detail.

First the linear Boltzmann operator in certain Banach spaces is rigorously defined, and then the existence of the Møller operators is proved by use of the theorem of Cook-Jauch-Kuroda, that is generalized to the case of a Banach space.

§ 1. The Abstract Cauchy Problem of Linear Transport Theory

In Statistical Mechanics transport phaenomena of neutrons and photons are decribed by the linear Boltzmann equation ([1–4]). One has to start from the formal Cauchy problem:

$$\frac{\partial n}{\partial t} = -v\,\mathrm{grad}_x n + \int_{R^3} k(x, v', v) n(x, v', t) dv' - \sigma(x, v) n \tag{1.1}$$

$$n(x, v, 0) = f(x, v).$$

x and v are three dimensional vectors: $x=(x_1, x_2, x_3)$ and $v=(v_1, v_2, v_3)$. $w=(x,v)$ is a six dimensional vector in the μ-space of Statistical Mechanics.

$n(w, t)$ is a real valued function with $\mathrm{dom}\, n = R^6 \times R$.

$k(x, v', v)$ and $\sigma(x, v)$ are non-negative, bounded and measurable functions with $\mathrm{dom}\, k = R^9$ and $\mathrm{dom}\, \sigma = R^6$. Both functions vanish for $x \in R^3 \sim D$, where D is a compact and convex subset of R^3. In neutron transport theory D stands for reactor and in radiation transfer theory for star. In transport theory sometimes $k(x, v' \to v)$ is preferred to $k(x, v', v)$. Physically $k(x, v', v)$ is the number of particles with final velocity v, that are generated after one particle with initial velocity v' has suffered a collision in x. In neutron transport theory $\sigma(x, v)$ has the meaning of a reaction rate, it actually equals $|v| \cdot \Sigma_t(x, v)$, $\Sigma_t(x, v)$ being the total macroscopic cross section. $\sigma(x, v)$ has the dimension of an inverse time. Later we also need

$$\sigma_s(x, v) := \int_{R^3} k(x, v, v') dv'. \tag{1.2}$$

Note that $k(x, v, v')$ appears in (1.2), but that $k(x, v', v)$ appears in (1.1)!

The formal Cauchy problem can also be written in the following form:

$$\frac{\partial n}{\partial t} = -v\,\mathrm{grad}_x n + \chi_D(x) \cdot \left[\int_{R^3} k(x, v', v) n(x, v', t) dv' - \sigma(x, v) n\right], \tag{1.3}$$