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Euclidean Field Theory

I. The Moment Problem

John L. Challifour and Steven P. Slinker
Department of Physics, Indiana University, Bloomington, Indiana, USA

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Abstract. The extension of the Schwinger functions to various positive linear functionals on the Borchers algebra is discussed. In one case, we construct a measure on $\mathscr{S}'$ and give criteria for uniqueness as well as for the homogeneous chaos to lead to an $\mathscr{L}_2$-space.

1. Introduction

In recent years there has been much development in constructive quantum field theory involving the formal framework of generalized random processes; see for example [5, 9, 19, 20]. Still unanswered is the question as to those Wightman quantum field theories admitting such a representation. In this paper we wish to begin an examination of this question and its consequences. More generally one may ask whether Nelson's sharp time euclidean framework [9] may be modified to encompass all Wightman theories or failing this, can restrictions on the latter be given for a reasonably broad equivalence theorem between relativistic and euclidean fields? The success of euclidean methods in constructive field theory amply warrants such an investigation in spite of the lack of a four dimensional example.

The first task of placing the relativistic theory within a euclidean framework has been completely solved by Osterwalder and Schrader [1, 5] in terms of Schwinger functions. The ideas presented here develop the point of view that the probabilistic euclidean field theory arises when these Schwinger functions admit certain extensions to the Borchers algebra over the underlying test function space. As a model, we study the Schwartz space on R^4 though it is clear any other complete nuclear *-algebra will serve equally well. Within this paper, we assume and examine two classes of extensions, the positive and strongly positive ones [10]. For the first case, a euclidean field theory results without necessarily requiring the existence of an infinite volume probability measure. Such a measure arises upon formulating a moment problem using the second notion of positivity. In the study of the $P(\Phi)_2$ model, extended Schwinger functions are defined through the infinite volume limit and positivity is an immediate consequence of the form for the finite volume measure. General mathematical conditions allowing existence of these extensions may be written down, but whether these require further restrictions beyond the Wightman axioms is not yet known.

Conditions on the relativistic theory which lead to most of Nelson's sharp time framework have been given by Simon [19]. More recently, starting with