Commun. math. Phys. 42, 195—208 (1975)

The Connexion of Duality and Causal Properties for Generalized Free Fields

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Received September 14, 1974

Abstract. It is shown that the time-slice axiom and the diamond property are equivalent for the generalized free field. If, in addition, there is a mass gap, duality is equivalent to either causality requirement. It is further shown that the local rings associated with certain space-time regions are factors in the case of causal generalized free fields with mass gap. Necessary and sufficient conditions for causality and duality and some examples for causal and acausal generalized free fields are also given.

Introduction

In their paper on the postulates of quantum field theory [27], Haag and Schroer considered, among other requirements, three restrictions on Wightman fields which they called "primitive causality", "Einstein causality" and "duality". The first two requirements are completeness postulates for the algebra of field operators associated with certain space-time regions:

A field is said to be *primitively causal* if the von Neumann algebra $R(S_\delta)$ generated by the field operators associated with an arbitrary time-slice S_δ of non-zero thickness δ already contains all field operators:

$$R(S_\delta) = R(M), \quad \text{all} \quad \delta > 0, \tag{0.1}$$

where M denotes Minkowski space. If (0.1) is valid, it is also said that the time-slice axiom holds (see Fig. 1).

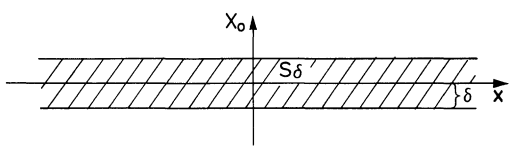

Fig. 1

A field is said to exhibit *Einstein causality* if the von Neumann algebra of field operators associated with an arbitrary cylinder $Z_{a,\varepsilon}$ contains all field operators associated with the double cone sustended by $Z_{a,\varepsilon}$:

$$R(Z_{a,\varepsilon}) = R(Z''_{a,\varepsilon}) \quad \text{all} \quad a > 0, \quad \text{all} \quad \varepsilon > 0. \tag{0.2}$$