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Schwinger Functions for the Yukawa Model in Two Dimensions with Space-Time Cutoff*

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Abstract. It is shown that a Euclidean version of the formulae of Matthews and Salam for the Green's functions of a two-dimensional Yukawa model with interaction in a finite space-time volume makes sense, if renormalized correctly.

1. Introduction

Twenty years ago Matthews and Salam [1] presented formulae for the Green's functions of a Yukawa theory which arose from formally integrating out the fermions in Feynman's "sum over histories" [2]. Those formulae expressed the Green's functions as functional integrals over the Bose field of certain combinations of determinants.

In this paper we show that a Euclidean version of those formulae for a two-dimensional Yukawa ("Y_2") model with a space-time cutoff for the interaction actually makes sense, if the necessary renormalization is carried out in the appropriate way.

We do not discuss here the problem of removing the space-time cutoff which would hopefully lead to Schwinger functions fulfilling all the axioms of Osterwalder and Schrader [3, 4] guaranteeing the existence of the corresponding quantum field theory fulfilling Wightman's axioms.

Recently Osterwalder and Schrader [5] have constructed Euclidean Fermi fields and proved a Feynman-Kac type formula which relates Euclidean expressions in a finite space-time valume (and containing ultraviolet cutoffs) to quantities of the "physical" Y_2 theory. This gives a connection between our integrability statements and the semiboundedness of the Y_2 Hamiltonian which was first proved by Glimm [6] (see also Glimm and Jaffe [7], Schrader [8], Brydges and Federbush [9], and Brydges [10]). But we are not able to construct an independent proof of this semi-boundedness at this stage; this would involve the removal of at least the time cutoff.

Osterwalder's and Schrader's fields can also be used to derive an ultraviolet and space-time cutoff version of the Matthews-Salam formula (1.1) which we take here simply as a heuristic starting point. This derivation also guarantees the physical ("Osterwalder-Schrader") positivity of the approximate Schwinger functions (see [5]) for suitably chosen space-time cutoff.

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