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Relativity Groups in the Presence of Matter

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Abstract. We show that the action of the boosts on an infinite system can be described continuously by bundle maps of Hilbert bundles based on the manifolds G/G_0, where G is the full relativity group and G_0 its closed subgroup which can be unitarily implemented on the fibre, which is a Hilbert space. We then develop a general theory of group representations on product bundles $M \times \mathscr{H}$, where M is a manifold and $\mathscr{H}$ a Hilbert space. We construct certain bundle representations of the Galilei and the Poincaré group and find that they correspond to various classes of elementary excitations. In particular, we define nonrelativistic zero-mass systems and obtain an analogue of the Faraday effect for the passage of hot electrons through matter. Our construction remains valid for the case when G_0 is the product of a lattice translation group and the time translations. We conclude that many qualitative features of the physics of condensed matter can be directly understood in terms of relativity group action on a bundle space as state space, which also suggests some avenues for further work.

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§ 0. Introduction

In this article we will determine how the full Galilei or Poincaré group acts on an infinite system[1]. It is known that the boost operations on such systems cannot be unitarily implemented in a reasonable Hilbert space. It will turn out that the state space is a Hilbert bundle and the boost operations correspond to bundle maps which carry the fibres continuously into other fibres. A point on the base space describes a "state" of the medium or the background. We will then investigate, by an appropriate theory of group representations, the "elementary" quantum-mechanical objects in a "medium".

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[1] By an infinite system we mean a system which is composed of an infinite number of massive particles.