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Analyticity Properties of the Correlation Functions for the Anisotropic Heisenberg Model

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Abstract. It is shown for the Heisenberg model that the correlation functions are analytic in h and T if $\mathrm{Re}(h) \neq 0$ and T is positive.

Introduction

The analyticity properties of the Ising model, when there is no phase transition, were established by Lee and Yang [6, 11] and by Lebowitz and Penrose [5]. The theorem of Lee-Yang about the zeros of the partition function of the system plays a prominent part in these papers. The generalization of this famous theorem to the case of the Heisenberg model was made by Asano [1] and Suzuki-Fisher [10]. With the help of this generalization we obtain analogous results as those obtained by Lebowitz and Penrose for the Ising model: the correlation functions are analytic in h and T if $\mathrm{Re}(h) \neq 0$ and T is positive. The proof follows closely that of Lebowitz and Penrose. We use essentially the theorem of Lee-Yang and the technique introduced by Asano [1]. Our proof is only valid if the total magnetization commutes with the Hamiltonian, and does not extend to the general case considered by Suzuki and Fisher [10].

Notation and Definition of the Model

The model is defined on the lattice $\mathbb{Z}^\nu$. With each point of the lattice we associate a spin $-1/2$, which we describe by a Hilbert space $\mathscr{H}_i$ isomorphic to $\mathbb{C}^2$, and by the Pauli matrices σ_i^x, σ_i^y, σ_i^z. We consider first a system restricted to a finite subset Λ of $\mathbb{Z}^\nu$. The corresponding Hilbert space is $\mathscr{H}_\Lambda = \bigotimes_{i\in\Lambda} \mathscr{H}_i$ and we choose the Hamiltonian as follows:

$$H_\Lambda = -\sum_{\substack{i \neq j \\ i,j\in\Lambda}} H(i,j) + h \sum_{i\in\Lambda} (\sigma_i^z + 1) \tag{1}$$

with

$$H(i,j) = K(i-j)\,(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) + J(i-j)\,\sigma_i^z \sigma_j^z\,. \tag{2}$$

In this formula $H(i,j)$ describes an interaction between two spins. The interaction will be a ferromagnetic one:

$$J(x) = J(-x) \geqq 0\,, \qquad K(x) = g(x)\,J(x) \tag{3a}$$