Commun. math. Phys. 41, 65—78 (1975)

Singularities in Globally Hyperbolic Space-Time

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Received April 7, 1974

Abstract. A singularity reached on a timelike curve in a globally hyperbolic space-time must be a point at which the Riemann tensor becomes infinite (as a curvature or intermediate singularity) or is of type D and electrovac.

1. Introduction

It is known ([1] § 8.2) that, in certain physically realistic situations, general relativity predicts either the occurrence of a singularity in spacetime or the violation of some sort of causality condition. Hitherto little has been known as to the nature of the singularities which might arise in this context: in particular, it has not been known whether or not the Riemann tensor must become "infinite". The aim of the present paper is to show that, in the situations envisaged in the singularity theorems, the Riemann tensor cannot be well-behaved.

By 'singularity' I mean a point p on the b-boundary $\dot{M}$ of a space-time M [2]. Such a point is the end-point of a curve in M which has finite length according to a generalised affine parameter defined by a parallely propagated (p.p.) tetrad (see § 2). In particular, the singularities predicted by the singularity theorems are the end-points of incomplete timelike geodesics. The precise construction of the b-boundary defines when two finite-length curves have the same end point, which enables one to divide singularities into three classes, as follows ([4], with slight changes).

(i) Curvature singularity: there is a curve running to the singularity on which $R_{\alpha\beta\gamma\delta}$ does not tend to a limit, in whatever tetrad it is evaluated.

(ii) Intermediate singularity: not (i); but there is a curve on which $R_{\alpha\beta\gamma\delta}$ does not tend to a limit in a p.p. tetrad.

(iii) Locally extensible: not (i) or (ii).

The justification for basing the classification on p.p. tetrads, and the reason for the name in (iii), lie in the result [5] that a curve on which $R_{\alpha\beta\gamma\delta}$ *does* tend to limit in a p.p. tetrad has a neighbourhood isometric to a neighbourhood in a singularity-free space time. If p is a singularity of this third type, then there are two sub-cases.

(a) p is inessential: there is an isometry $\psi : M \to M'$ into a larger (C^2, Hausdorff) space-time which carries p into an interior point, i.e. $\overline{\psi} p \in M'$ (where $\overline{\psi}$ is ψ extended to the b-boundary, [3]).

(b) Otherwise p is essential.

An inessential singularity is one created by "cutting out" a portion from a larger space-time M'. There would seem to be no conceivable physical mechanism